

Dmitri Alekseevsky

Pseudo-Riemannian manifolds with many Killing spinors

The talk is based on joint work with V. Cortés. We prove that a pseudo-Riemannian spin manifold with sufficiently many linearly independent Killing spinors with the same Killing number (respectively, conformally Killing spinors) are locally homogeneous (respectively, conformally locally homogeneous). Sufficiently many means more than $(1/2)N$ in Riemannian case and more than $(3/4)N$ for most other signatures, where N is the rank of the spinor bundle. We give also an inductive construction of Riemannian manifolds admitting a Killing spinor.

Lars Andersson

Hidden symmetries and the wave equation on Kerr

The wave equation on the Kerr spacetime is interesting as a toy model for the problem of stability of the Kerr black hole. In analyzing waves on Kerr one encounters two important difficulties. The Kerr spacetime has only two Killing fields, corresponding to stationarity and axial symmetry. Further, the “photon sphere”, i.e., the region filled with rotating null geodesics, has codimension zero, which makes trapping a serious problem. In recent work with Pieter Blue, we have been able to circumvent these difficulties and give a “physical space” approach to estimates for the wave equation, by making use of the hidden symmetry of the Kerr spacetime, discovered by Carter. By utilizing the fact that an operator related to the Carter constant commutes with the Kerr wave operator, we are able to prove energy bounds, trapping, and dispersive estimates for the wave equation on Kerr.

Christian Bär

Dirac type operators on Lorentzian manifolds

Jürgen Berndt

Cohomogeneity one actions on symmetric spaces of noncompact type

An isometric action of a Lie group on a Riemannian manifold is of cohomogeneity one if the corresponding orbit space is one-dimensional. In the talk I plan to present an introduction to cohomogeneity one actions and to discuss some classifications of cohomogeneity one actions on symmetric spaces.

Alexey Bolsinov

Projectively equivalent pseudo- Riemannian metrics and integrable

$so(p, q)$ -tops

Two (pseudo-)Riemannian metrics are called projectively equivalent, if they have the same geodesics considered as unparametrized curves. The purpose of the talk is to discuss a relationship between the theory of projectively equivalent pseudo-Riemannian metrics and the theory of integrable systems on semisimple Lie algebras. One of the most important examples of such systems was discovered by S.Manakov in 1976. From the algebraic point it can be simply viewed as a self-adjoint operator on $so(n)$ satisfying certain remarkable properties. It turns out that these operators are essentially the same as the Riemann curvature tensors of the metrics admitting non-trivial projectively equivalent partners. This surprising observation turns out to be not just curious but also useful in both directions.

Piotr T. Chruściel

On higher dimensional black holes

I will describe the Emparan-Reall black rings and their maximal extensions. Some further results concerning other higher-dimensional black holes will be discussed. The talk will be based on joint work with Julien Cortier (<http://arxiv.org/abs/0807.2309>) as well as on work in progress with Julien Cortier, Michal Eckstein, A. Garcia Parrado and Sebastian Szybka.

Vicente Cortés

Half-flat structures and special holonomy

Andrzej Derdzinski

Geometric conditions leading to Patterson-Walker extension metrics

Patterson and Walker introduced in 1952 the class of so-called Riemann extension metrics, which are indefinite metrics of the neutral signature on $M = T^*S$, arising from any fixed connection on the manifold S and any fixed symmetric 2-tensor on S . Afifi (1954) found an intrinsic local characterization of such metrics. This talk presents some cases in which natural geometric conditions imposed of a metric g of the neutral signature $(- -++)$ on a four-manifold imply that, locally, g is a Patterson-Walker extension metric. One of these cases is provided by self-dual neutral Einstein four-manifolds of Petrov type III, which in addition have the Walker property (that is, admit a two-dimensional null parallel distribution compatible with the orientation). It is shown that such a manifold (M, g) cannot be compact or locally homogeneous, and its maximum possible degree of mobility is 3. The value 3 of the degree of mobility is realized only by a small class of metrics g , which can be described in terms of three-dimensional Lorentzian geometry.

Felix Finster

Causal variational principles in discrete and continuum space-times

Thomas Leistner

Conformal pp-waves and their ambient metrics

Vladimir Matveev

Projective transformation of pseudo-Riemannian manifolds: rigidity of Einstein manifolds and Lichnerowich conjecture

Two metrics g and \bar{g} are geodesically equivalent, if every g -geodesic, after the appropriate reparameterisation, is a \bar{g} -geodesic. In the present talk, which is mostly based on recent joint paper with Kiosak (<http://xxx.lanl.gov/abs/0806.3169>), I will consider geodesic equivalence of pseudo-Riemannian metrics such that the metric g is Einstein. The main result of the talk gives a complete answer to a question posed by Weyl and Petrov and is

Theorem: Let g is an Einstein metric on a 4-dimensional M . If \bar{g} is geodesically equivalent to g , then it is affine equivalent to g , i.e., g and \bar{g} have the same Levi-Civita connection.

The proof of this theorem is nontrivial and contains new ideas. The rest of the talk is

devoted to application of these new ideas to similar questions in pseudo-Riemannian geometry, with the hope that the participants of the summer school can further develop them.

Karin Melnick

Nilpotent groups of conformal flows on pseudo-Riemannian manifold

By a celebrated theorem of Lelong-Ferrand, proving a conjecture of Lichnerowicz, a compact Riemannian manifold with noncompact conformal group is conformally equivalent to the round sphere. The "pseudo-Riemannian Lichnerowicz conjecture," that a compact pseudo-Riemannian manifold (M, g) with essential conformal group is conformally flat, remains open. I will present a tight upper bound on the nilpotence degree of a connected nilpotent subgroup of $\text{Conf}(M, g)$ and a theorem that if this maximal degree is attained, then M is conformally equivalent to a quotient of the Einstein space. These results support the pseudo-Riemannian Lichnerowicz conjecture. Joint work with Charles Frances.

Marc Nardmann

Some global Ricci and scalar curvature problems in Lorentzian geometry

Martin Olbrich

Non-semisimple variants of classical pseudo-Riemannian symmetric spaces

Carlos Olmos

Submanifold geometry and Berger-type theorems

The normal holonomy of a submanifold of a space form, turns out to be even simpler than Riemannian holonomy (this is also true for the Lorentzian case, from a recent result of K. Lrz). This has interesting consequences not only in submanifold geometry, but also in Riemannian geometry. In fact, the Berger holonomy theorem depends strongly on the fact that the normal holonomy has a very special form. In this talk we would like to draw the attention on some results, similar to that of Berger, in the context of submanifold or Riemannian geometry (that also depend on the special form of the normal holonomy). Finally, we will discuss some applications to homogeneous geometry which in particular explains the inextendibility of isotropy irreducible spaces (in the sense of Wolf and Wang-Ziller).

Alan Rendall

The Einstein-Maxwell equations and the complex hyperbolic plane

The subject of this talk is the long-time dynamics of certain solutions of the Einstein equations coupled to the Maxwell equations. I will start by discussing the definition of the equations and the motivation for considering solutions with symmetry. I will then discuss a particular kind of symmetry called Gowdy symmetry. This leads to equations for a mapping from a two- or three-dimensional Lorentzian manifold into the real hyperbolic plane (for vanishing Maxwell field) or the complex hyperbolic plane (for a general Maxwell field). The central theme of the talk is the interplay between the study of solutions of a hyperbolic system of partial differential equations and the differential geometry of the target manifold where the solutions take their values.

Short talks

Marie-Amélie Lawn

Pluriminimal immersions of pseudo-Riemannian manifolds into indefinite euclidean spaces

Kishore Marathe

Generalized gravitational instantons

Katja Sagerschnig

Conformal structures associated to generic rank 2 distributions in dimension 5.

It is a classical result of E. Cartan that maximally non-integrable distributions of rank 2 on 5-dimensional manifolds can be described as parabolic geometries. Based on Cartan's work, P. Nurowski associated to such a distribution a natural conformal class of pseudo-Riemannian metrics of signature (2,3). In this talk we shall show that methods available for parabolic geometries can be applied to the study of these conformal structures. For example, we obtain a characterization in terms of certain normal conformal Killing 2-forms and we can prove that the space of conformal Killing fields decomposes into symmetries of the distribution and almost Einstein scales. The constructions are analogous to known ones for Fefferman spaces. This is joint work with M. Hammerl.

Mike Scherfner

Geometrical Aspects of Time-Machines

From the mathematical point of view time-machines show up as closed timelike curves in Lorentzian 4-manifolds. We will present some of the preliminaries for the understanding of this topic and discuss several types of causality. Furthermore we give recent results and their visualizations.

Jürgen Tolksdorf

Real bi-graded Clifford modules, the Majorana equation and the Standard Model Action

The fundamental grading involution that underlies the Dirac equation is provided by parity. In contrast, the Majorana equation is based upon charge conjugation. Together, these two grading involutions form what is called a Majorana module. On these modules there exists a natural class of Dirac operators encoding the full action functional of the Standard Model of particle physics (including gravity).